5.3b Definite Integrals and Antiderivatives

Rules for Definite Integrals

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int_b^c f(x)\,dx = \int_a^c f(x)\,dx - \int_a^b f(x)\,dx$$

$$\int k\cdot f(x)\,dx = k\cdot\int_a^b f(x)\,dx$$

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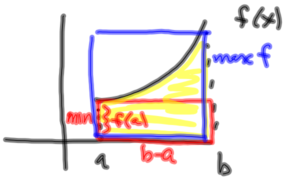
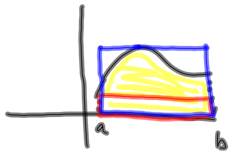
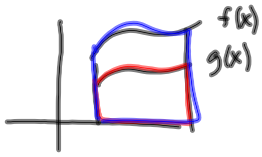
min max inequality





Domination

$f(x) \geq g(x)$



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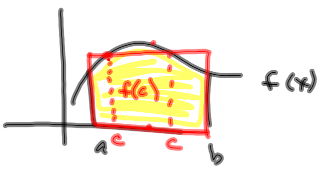
Average (Mean) Value



$$f(c)(b-a) = \int_a^b f(x)\,dx$$

$$f(c) = \boxed{\frac{1}{b-a}\int_a^b f(x)\,dx}$$

ave value of f(x)

MVT says "c" exists

$a<c<b$

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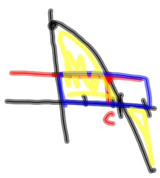
Find the average value of f(x) = 4-x^2 on [0,3]. Does *f* actually take on this value at some point on the given interval? If so, where?

find c

$$\bar{y} = \frac{1}{3-0}\int_0^3 4-x^2\,dx = 1$$



$4-x^2 = 1$

$x = \cancel{\pm}\sqrt{3}$

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Mean Value Theorem for Definite Integrals

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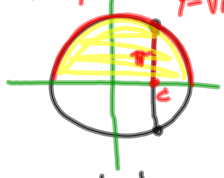
How long is the average chord of a circle of radius 4? Find the value that satisfies the Mean Value Theorem for Definite Integrals.

$x^2+y^2=16$ $\quad y=\sqrt{16-x^2}$



ave chord: 2π

$$\bar{y} = \frac{1}{4-{-4}}\int_{-4}^{4}\sqrt{16-x^2}\,dx = \pi$$

$$= \frac{1}{8}\cdot\frac{\pi\cdot 4^2}{2} = \pi$$

$$\pi = \sqrt{16-x^2}$$

$$\pi^2 = 16-x^2$$

$$x^2=\sqrt{16-\pi^2} = 2.475$$

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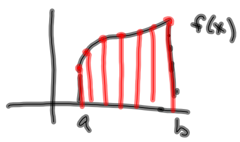
$y=f(x)$ on $[a,b]$



$$\frac{f(x_1)+f(x_2)+\dots+f(x_n)}{n}$$

$\Delta x = \frac{b-a}{n}$ $\quad$ approx: $\frac{\sum_{i=1}^{n} f(x_i)}{n}$

$\frac{\Delta x}{b-a} = \frac{1}{n}$

$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$

$\int_a^b f(x)\,dx$

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\frac{1}{n}$$

$$\lim_{n\to\infty}\sum f(x_i)\,\frac{\Delta x}{b-a}$$

$$\frac{1}{b-a}\lim_{n\to\infty}\sum f(x_i)\,\Delta x$$

$$\bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

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